Proyecto de Tesis de Maestría (Miguel Bernal)

Título: Estudio de sistemas no lineales por medio de la relación entre estructuras convexas, desigualdades matriciales lineales y estabilidad de polinomios.

Problema a resolver: Considere sistemas no lineales afines en el control reescritos en forma convexa por medio de la técnica de *sector no lineal* [1]:

$$\dot{x}(t) = A(x(t))x(t) + B(x(t))u(t) = \sum_{i=1}^{r} h_i(x)\left(A_i x(t) + B_i u(t)\right), \tag{1}$$

donde $x(t) \in \mathbb{R}^n$ es el estado, $u(t) \in \mathbb{R}^m$ es la entrada de control, $A(\cdot)$ y $B(\cdot)$ son funciones matriciales suficientemente suaves, posiblemente no lineales, acotadas en un conjunto compacto $\Omega \subset \mathbb{R}^n : 0 \in \Omega$, $h_i(x)$ son funciones *convexas* denominadas *funciones de pertenencia o membresía* que satisfacen $\sum_{i=1}^{r} h_i(x) = 1$, $0 \leq h_i \leq 1$ en el compacto Ω, A_i y B_i, $i \in \{1, 2, \ldots, r\}$ matrices "extremos" del politopo que representa *exactamente* al sistema no lineal de la izquierda mediante el modelo convexo de la derecha en Ω. Estos modelos suelen denominarse modelos Takagi-Sugeno (TS) o quasi-LPV (linear parameter varying) [2].

La estabilidad del sistema no lineal reescrito en forma convexa (1) con $u(t) = 0$ así como la estabilización por medio de la ley de control *parallel distribution compensation* (PDC) $u(t) = \sum_{i=1}^{r} h_i(x)F_i x(t)$ ha sido ampliamente analizada por medio del método directo de Lyapunov [3] que conduce, tratado adecuadamente, a desigualdades matriciales lineales (LMIs) [4]. Las LMIs son de interés porque, (a) al poseer un óptimo global, pueden resolverse computacionalmente en *tiempo polinomial* y (b) son matemáticamente *decidibles* en relación con su factibilidad (es decir, puede saberse siempre si tienen solución o no la tienen) [5].

Sin embargo, las condiciones de estabilidad y estabilización de sistemas no lineales por medio de su representación convexa TS son, hasta el momento, sólo *suficientes*, pero *no necesarias*. Esto significa que hay sistemas estables o estabilizables cuya estabilidad o estabilización no puede establecerse por estos medios, a pesar de que (a) se han utilizado familias más grandes de funciones de Lyapunov como las *piecewise* [6], [7], las *convexas* [8], [9] o las *polinomiales homogéneas* [10], [11], [12], (b) se han utilizado relajaciones de suma convexa que son asintóticamente suficientes y necesarias [13], (c) se han utilizado modelos convexos más inclusivos como los descriptores [14] o polinomiales [15].

Ahora bien, en el contexto de control robusto de sistemas lineales [16] existe un resultado debido a Kharitonov conocido como el teorema de los cuatro puntos [17] que establece que una familia de polinomios con coeficientes *en intervalos* $p(s) = [a_n, b_n]\, s^n + [a_{n-1}, b_{n-1}]\, s^{n-1} + \cdots + [a_1, b_1]\, s^1 + [a_0, b_0]$ será estable *si y sólo si* los cuatro polinomios de Kharitonov siguientes son estables (Hurwitz):

$$k_1(s) = a_n s^n + a_{n-1}s^{n-1} + b_{n-2}s^{n-2} + b_{n-3}s^{n-3} \cdots \quad k_2(s) = a_n s^n + b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + a_{n-3}s^{n-3} \cdots$$
$$k_3(s) = b_n s^n + b_{n-1}s^{n-1} + a_{n-2}s^{n-2} + a_{n-3}s^{n-3} \cdots \quad k_4(s) = b_n s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + b_{n-3}s^{n-3} \cdots$$

Numerosos trabajos se han derivado del teorema de Kharitonov en forma de extensiones y equivalencias [18], [20], [22], [21], [24], [25], [26]. Algunos trabajos han intentado explotar la relación de los sistemas definidos por intervalos de polinomios (como las familias de Kharitonov) y los sistemas definidos por intervalos de matrices o politopos, es decir, sistemas quasi-LPV o TS [19], [23], [27], [28].

El objeto de esta propuesta es estudiar la relación entre las condiciones *suficientes* de estabilidad y estabilización de sistemas convexos TS y las condiciones *suficientes y necesarias* para estabilidad de familias de polinomios. ¿Puede aprenderse algo a partir de Kharitonov para que ir de condiciones suficientes a necesarias en el contexto TS-LMI? Todavía más, el tema guarda una estrecha relación con sistemas con retardos naturales o inducidos [29], [30], [31], [32], [33], [34], [35], [36] y aplicaciones en diversos campos [37], [38].

Productos académicos comprometidos: 1 artículo de conferencia internacional arbitrada publicado y 1 artículo de revista indizada sometido, ambos antes del 31 de agosto de 2017.

Estancia del estudiante: En la UAM Iztapalapa (DF) con el Dr. Baltazar Aguirre (SNI I) y/o con el Dr. Raúl Villafuerte de la UAEH, 1-2 meses en 2017.

Conferencia del estudiante: International Conference on Electrical Engineering, Computing Science and Automatic Control (CCE 2016/2017) ó en su defecto el Congreso Nacional de Control Automático de la Asociación de México de Control Automático (AMCA 2016/2017).

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